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INFLUENCE OF UNSTEADY FORCES ACTING ON A PARTICLE IN A SUSPENSION APPLICATION TO THE SOUND PROPAGATION

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Abstract—First, the influence of the unsteady forces (the pressure gradient, the virtual mass effect and the Basset history term) on the complex velocities ratio of the fluid and of the dispersed phases has been studied. To this end, the particle momentum equation is linearized for small oscillating motion of the two phases which are at rest in the reference state. It is shown that the unsteady terms are of great importance when the coefficient χ, mass density of the particle divided by the mass density of the fluid, becomes small. A particular study of the Basset history term is also investigated. Then, a two fluids theory, including viscous and thermal losses effects, is developed for calculating the velocity and the damping of the sound propagating in a two-phase flow. As the former treatment, the classical equations of the multiphase flows are linearized and the dispersion equation of the acoustical wave is obtained. Several tendencies and the special part played by the Basset history term in acoustics are pointed out.

Key Words: multiphase flow, suspension, unsteady forces, sound propagation

INTRODUCTION

The unsteady terms in the expression of particles drag are often neglected in the equations of moving multiphase flows. When the flow motion is well established, the main force acting on a particle is the viscous Stokes drag whose expression is well-known for small Reynolds numbers. The Reynolds number is defined by: $\mathrm{Re} = dV_{\mathrm{rel}}/\nu$ where, d is the particle diameter, V_{rel} is the relative velocity between the fluid and the sphere far away from the particle and ν is the fluid kinematic viscosity. In this theory the particles are taken to be rigid and spherical and there is no interaction between them (Germain 1962; Fortier 1967). The flow is irrotational and the surface tension is supposed to have no influence. With these assumptions, the particle acceleration is proportional to the local relative average velocity between the fluid and the particle and a kinetic relaxation time appears (Kuentzmann 1973). This relaxation time τ_V depends on the coefficient of Stokes formula. It is also used in the expression of the entropy production rate for the flow when the Onsager linearized theory is applied (Prud'homme 1988).

Spherical rigid particles are not necessarily solid, some liquid droplets can remain spherical in hard conditions when their diameter and their capillarity number are small enough (Feuillebois 1991). If the particles are not of spherical shape, it is necessary to introduce a shape factor.

In Stokes flows, the only modification of τ_V may not be sufficient to take into account possible unsteady effects. The problem is no longer linear and other terms must be added to the Stokes drag. If the particle is rigid and if it does not rotate, these terms are: the pressure gradient, the virtual mass effect, the weight and the Basset history term. If the particles concentration is small and if the particles do not modify the velocity of the fluid, the two first terms are easy to evaluate. The Basset term is much more difficult to derive in the general cases (Rusanov 1953; Fortier 1967). Nevertheless, in a linear (small amplitude) oscillatory situation, the Basset integral easily reduces to a simple form (Landau & Lifshitz 1971). Experimental studies and more recently direct numerical simulations give a proper understanding of the different forces influences (Rivero *et al.* 1991).

The net force in Stokes flow ($\mathrm{Re} \ll 1$) is different from the net force at higher Reynolds numbers. Indeed, some coefficients must be introduced such as: the Reynolds number and the acceleration number (Clift *et al.* 1978; Rivero 1991).

In the first part of this paper, we study the respective influences of the previous terms on the ratio particle velocity/fluid velocity. The motion is oscillatory, time dependent and eventually space dependent, its amplitude being small ($p'_{max}/p_o \ll 1$, p'_{max} is the maximum pressure perturbation and p_o is the pressure in the reference state). The linearization of the particle dynamical equation leads to the expression of the complex velocities ratio for the particle and the fluid as a function of the product $\omega\tau$ (ω is the angular frequency, τ the time defined by τ_V/χ and χ the mass density ratio ρ_{ps}/ρ_{gs}). The study in the complex plane shows that the ratio of the velocities moduli is sometimes very different from unity and the phase-lag between the two velocities is not always negligible.

In the second part, a theory is developed, on the base of a two fluids model, for studying the influences of the different forces on the propagation and the damping of sound in two-phase flows. In elastic media (water, air, etc.), the sound propagates nearly without damping. That is no longer the case for porous, soft or multiphase media (Matras 1972). Indeed, the exchanges of momentum and energy between the two phases yield energy loss effects.

The propagation of sound has been investigated by many authors. A part of the published papers is relating to sound propagation in bubbly flows (Hinze 1975; Clift *et al.* 1978; Biesheuvel & Wijngaarden 1984; Wijngaarden & Kapteyn 1990; Sangani *et al.* 1991). These differ in their approach compared to the present paper which is relating to condensed particles in a fluid phase. Gregor & Rumpf (1975), using mass and momentum balances, show that the velocity of sound depends on the relative velocity between the two phases, on the ratio of densities, on the particles concentration, on the particle diameter, on the drag coefficient and on the frequency of sound. Some authors express the sound velocity using a thermodynamic method (Michaelides & Zissis 1983). Allegra & Hawley (1972) introduce wave equations (compressional, thermal and viscous waves). Some studies use a method of linearization with introducing a complex wave-number (Atkinson & Kytomaa 1992), that is the kind of approach which is chosen here.

In this second part, the considered fluid is a gas. The compressibility effects and the thermal exchanges are no more negligible. These thermal exchanges are characterized by a thermal relaxation time called τ_t. We use the former treatment but now the small oscillating perturbations are explicitly time and space dependent. The model requires the wave-length to be large compared to the particle radius: $r_a \ll \lambda$. Indeed, if this inequality is not satisfied, many others phenomena occur such as the wave reflection on the particle. Furthermore, the physical properties would not be uniform at the surface of the sphere. Let us introduce the acoustical Reynolds number: $\mathrm{Re}\, c = cr_a/\nu$. As $c = f\lambda$ (c is the sound speed and f is the frequency of the acoustical wave), the condition $r_a \ll \lambda$, may be written: $\tau\omega \ll \mathrm{Re}\, c$.

Throughout the calculations, the following general assumptions are made. The studied medium consists of a two-phase suspension: a fluid phase (newtonian fluid) and a dispersed phase. The subscripts "g" and "p" stand respectively for the fluid and the particles. The particles distribution is supposed to be statically homogeneous so that the isotropy condition is satisfied. The gas volumic fraction, called ϵ, is nearly equal to unity in the case of a suspension. The particles occupy a small volume in the mixture, hence the interactions terms between them are overlooked (distance effects, collisions). We admit that a statistical study of the suspension is feasible and it is assumed that the particles as well as the fluid phase constitute a continuum. Finally, we suppose that the particles have all the same diameter and that the volume forces, except the gravity and the Archimedes force, are negligible.

PART 1. EQUATION OF MOTION FOR THE PARTICLES AND LINEAR ANALYSIS OF VELOCITIES FOR SMALL OSCILLATING PERTURBATIONS

Let us define a kinetic relaxation time as being the ratio between the particle momentum in the fluid frame and the modulus of the fluid–particle interaction force: $\mathbf{f}_p$ (Lupoglazoff 1989).

$$\tau_v = \frac{m_p v_{rel}}{\|\mathbf{f}_p\|} \qquad [1]$$

where m_p is the mass of the particle.

Thus, if τ_v is large compared to the characteristic time T of the fluid motion, then the average motion of the dispersed phase is slightly influenced by the local conditions of the flow (vortices, vibrations, etc.). But if τ_V is much smaller than T, then the particles cloud motion follows perfectly the fluid flow.

In a monodisperse suspension, the simplified motion equation, without body forces, is:

$$\frac{d_p \mathbf{v}_p}{dt} = \frac{\mathbf{v}_g - \mathbf{v}_p}{\tau_v} \qquad [2]$$

where the $d_p(\ )/dt$ term denotes the material time derivative following the moving sphere with the velocity $\mathbf{v}_p$: $d_p(\ )/dt = \partial(\ )/\partial t + \mathbf{v}_p \cdot \overrightarrow{\text{grad}}(\ )$. Some more complete expressions, valid for low Reynolds number, take into account other effects and the final equation for the momentum interaction force takes the form (Tchen 1947; Corrsin & Lumley 1956; Hinze 1975; Clift *et al.* 1978; Maxey & Riley 1983):

$$\frac{4}{3}\pi r_a^3 \rho_{ps} \frac{d_p \mathbf{v}_p}{dt} = \underset{(a)}{6\pi\mu r_a(\mathbf{v}_g - \mathbf{v}_p)} + \underset{(b)}{\frac{2}{3}\pi r_a^3 \rho_{gs}\left(\frac{d_p \mathbf{v}_g}{dt} - \frac{d_p \mathbf{v}_p}{dt}\right)}$$

$$\underset{(c)}{-\frac{4}{3}\pi r_a^3 \overrightarrow{\text{grad}}(p)} + \underset{(d)}{6 r_a^2 \sqrt{\pi \rho_{gs} \mu} \int_0^t \frac{\left(\frac{d_p \mathbf{v}_g}{dt'} - \frac{d_p \mathbf{v}_p}{dt'}\right)}{\sqrt{t - t'}} dt'} + \underset{(e)}{\frac{4}{3}\pi r_a^3 \rho_{ps} \mathbf{g}}, \qquad [3]$$

the letters (a)–(e) referring to the different terms of the second member.

As mentioned above, the particle does not rotate. Equation [3] does not contain any interparticular pressure term since all particles interactions are neglected. Note that more sophisticated expressions contain other forces, such as the Faxen term (Gatignol 1983) or take into account the effect of compressible external flow on the added-mass term (Maxey & Riley 1983).

Meaning of the different terms of [3]

First member: acceleration of the particle.

Second member:

(a) Stokes drag. For a spherical rigid particle of diameter d, if the density and the viscosity of the fluid are constant and if the inertia forces are negligible compared to the viscosity forces, that is to say: $\text{Re} = dV_{rel}/\nu \ll 1$, then the Stokes drag force is equal to:

$$\mathbf{f}_v = 6\pi\mu r_a(\mathbf{v}_g - \mathbf{v}_p) \qquad [4]$$

where μ is the fluid dynamic viscosity, (g) and (p) are related respectively to the fluid and the particles in the mixture.

The coefficient τ_v, previously introduced, has the following value:

$$\tau_v = \frac{2}{9}\rho_{gs}\frac{r_a^2}{\mu} \qquad [5]$$

where ρ_{gs} is the specific mass density of the fluid.

Fortier (1967) shows that:

$$\frac{|\text{inertia forces}|}{|\text{Stokes drag}|} < \text{Re}/6$$

If $\text{Re} > 0.1$ the inertia forces can no more be neglected. Moreover, if Re increases then the unsteady and dissymmetric characters of the flow become important. When Re is very large, $\mathbf{f}_p$ may be written as a bounded expansion of increasing powers of Re.

The drag force for a spherical particle constituted of a fluid viscosity μ_2 put inside a fluid of viscosity μ_1 is (Fortier 1967):

$$\mathbf{f}_v = 4\pi r_a \mu_1 \frac{2 + 3\psi}{2 + 2\psi}(\mathbf{v}_g - \mathbf{v}_p) \quad \text{with} \quad \psi = \mu_2/\mu_1 \qquad [6]$$

For a liquid droplet in a gas: $\psi \to \infty$: $\mathbf{f}_v = 6\pi r_a \mu_l (\mathbf{v}_g - \mathbf{v}_p)$. And for a bubble in a liquid: $\psi \to 0$: $\mathbf{f}_v = 4\pi r_a \mu_l (\mathbf{v}_g - \mathbf{v}_p)$.

(b) Virtual mass effect. The force exerted by an incompressible perfect fluid at rest on a isolated sphere moving with the unsteady velocity $\mathbf{v}_p$ is:

$$\mathbf{F} = -C\tfrac{4}{3}\pi r_a^3 \rho_{gs} \frac{d_p \mathbf{v}_p}{dt} \qquad [7a]$$

C is the added mass coefficient. For a spherical rigid and isolated particle: $C = 1/2$. This force means that a virtual mass must be added to the particle. If the fluid velocity is $\mathbf{v}_g$, then the force imparted by the fluid on the sphere is given by:

$$\mathbf{F} = -C\tfrac{4}{3}\pi r_a^3 \rho_{gs} \left(\frac{d_p \mathbf{v}_p}{dt} - \frac{d_p \mathbf{v}_g}{dt}\right) \qquad [7b]$$

A more complete theory shows that C is a second order tensor (Drew *et al.* 1979). If the flow is locally isotropic C is a scalar. Other studies show that C depends on the particles concentration and on the geometrical configuration of the suspension. Its value decreases if the number of particles increases. For a random distribution of monodisperse particles, $C = c/2$ (Atkinson & Kytomaa 1992). C seems to be independent on the Reynolds number and on the acceleration number whose expression is:

$$A_C = V_{rel}^2/(2r_a a), \quad \text{with} \quad a = \left\| \frac{d_p \mathbf{v}_p}{dt} - \frac{d_p \mathbf{v}_g}{dt} \right\| \qquad [8]$$

Since we suppose that the net force exerted on the particle is a sum of several terms of different and independent physical origins, [7b] may be used for the total force applied to one or several particles. Some authors find some expressions which are sometimes a bit different. Drew *et al.* (1979), saying that the virtual mass force must be objective, agree with the previous expression in the case of a spherical droplet accelerated in a quiescent fluid. But they find a result with an opposite sign for a spherical bubble moving in a liquid at rest. Nevertheless, we use the classical result of [3] which is used by many authors.

(c) Pressure gradient. The pressure gradient is supposed to be uniform around the particle. The resultant force is:

$$\mathbf{F} = -\int_{\mathfrak{s}} p\mathbf{n}\, dS = -\int_{\mathfrak{v}} \overrightarrow{\text{grad}}\,(p)\, dV = -\overrightarrow{\text{grad}}\,(p)\tfrac{4}{3}\pi r_a^3 \qquad [9a]$$

$\mathfrak{s}$ is the surface sphere, $\mathfrak{v}$ its volume and $\mathbf{n}$ is the unit outward normal.

If we suppose, in first approximation, that the presence of the particles does not modify the flow locally, then the pressure gradient can be derived from the momentum equation of the fluid:

$$-\overrightarrow{\text{grad}}\,(p) = \rho_{gs} \frac{d_g \mathbf{v}_g}{dt} - \rho_{gs} \mathbf{g} \qquad [9b]$$

where $\mathbf{g}$ is the gravity acceleration vector.

The local velocity of the fluid being $\mathbf{v}_g$, we have: $d_g(\ )/dt = \partial(\ )/\partial t + \mathbf{v}_g \cdot \overrightarrow{\text{grad}}\,(\ )$.

Thus:

$$\mathbf{F} = \tfrac{4}{3}\pi r_a^3 \rho_{gs} \left(\frac{d_g \mathbf{v}_g}{dt} - \mathbf{g}\right) \qquad [9c]$$

(d) Basset history term. Some complicated algebra (Maxey & Riley 1993) gives the following expression:

$$\mathbf{F} = C_H r_a^2 \sqrt{\pi \nu \rho_{gs}} \int_0^t \frac{\left(\dfrac{d_p \mathbf{v}_g}{dt'} - \dfrac{d_p \mathbf{v}_p}{dt'}\right)}{\sqrt{t - t'}}\, dt' \qquad [10]$$

This term represents the history of the viscous effects on the particle. $C_H = 6$ when $\text{Re} \ll 1$ and $A_C \ll 1$ ([8]).

(e) Weight of the particle.

Now, let us note:

$$\rho_{ps}/\rho_{gs} = \chi \quad \text{and} \quad \tau = \tfrac{2}{9}\rho_{gs}\frac{r_a^2}{\mu}, \qquad [11]$$

χ being the ratio of mass densities ($\tau_v = \tau\chi$). Then the particle motion equation is:

$$\chi\tau\frac{d_p\mathbf{v}_p}{dt} = \mathbf{v}_g - \mathbf{v}_p + \frac{\tau}{2}\left(\frac{d_p\mathbf{v}_g}{dt} - \frac{d_p\mathbf{v}_p}{dt}\right) + \tau\frac{d_g\mathbf{v}_g}{dt} + \tau(\chi-1)\mathbf{g} + 3\sqrt{\frac{\tau}{2\pi}}\int_0^t \frac{\left(\frac{d_p\mathbf{v}_g}{dt'} - \frac{d_p\mathbf{v}_p}{dt'}\right)}{\sqrt{t-t'}}\,dt' \qquad [12a]$$

Projecting on the horizontal plane and admitting a one-dimensional motion, ($\mathbf{v}_p = v_p\mathbf{i}$, $\mathbf{v}_g = v_g\mathbf{i}$, $\mathbf{i}$ is a horizontal unit vector), the previous equation may be written:

$$\chi\tau\frac{d_pv_p}{dt} = v_g - v_p + \frac{\tau}{2}\left(\frac{d_pv_g}{dt} - \frac{d_pv_p}{dt}\right) + \tau\frac{d_gv_g}{dt} + 3\sqrt{\frac{\tau}{2\pi}}\int_0^t \frac{\left(\frac{d_pv_g}{dt'} - \frac{d_pv_p}{dt'}\right)}{\sqrt{t-t'}}\,dt' \qquad [12b]$$

To study the influence of the different terms of [3] on the particle motion, the velocities are perturbed from their steady state value by some small amount. The steady state being indicated by the subscript (o), the respective total velocities of the fluid and the particles are:

$$v_g = v_{go} + v'_g \qquad [13a]$$

$$v_p = v_{po} + v'_p \qquad [13b]$$

v'_g and v'_p are the velocities perturbations. Choosing a steady state at rest, it follows that: $v_{go} = 0$ and $v_{po} = 0$.

At the first order approximation, [12b] becomes:

$$\frac{\partial v'_p}{\partial t} = \frac{v'_g - v'_p}{\tau_v} + \frac{1}{2\chi}\left(\frac{\partial v'_g}{\partial t} - \frac{\partial v'_p}{\partial t}\right) + \frac{1}{\chi}\frac{\partial v'_g}{\partial t} + \frac{3}{\tau_v}\sqrt{\frac{\tau}{2\pi}}\int_0^t \frac{\left(\frac{\partial v'_g}{\partial t'} - \frac{\partial v'_p}{\partial t'}\right)}{\sqrt{t-t'}}\,dt' \qquad [14]$$

Some small periodic perturbations are chosen in the form:

$$v'_g = V_g\exp(i\omega t) \qquad [15a]$$

$$v'_p = V_p\exp(i\omega t) \qquad [15b]$$

(see also Hinze 1975, who represents v_g and v_p by a Fourier integral).

Since the flow is one-dimensional and incompressible, these terms are independent of the position. For a compressible fluid, V_g and V_p are position dependent. It should be noted here that [3] has been formulated for an incompressible fluid. Some authors have found semi-empirical formulae to take into consideration the compressibility of the fluid. Nevertheless, these corrections are not necessary for small perturbations (Kuentzmann 1973). So whether the fluid is compressible or not, we may write:

$$\underset{(a)}{i\omega\chi\tau V_p} = V_g - V_p + \underset{(b)}{i\omega\frac{\tau}{2}(V_g - V_p)} + \underset{(c)}{i\omega\tau V_g} + 3\sqrt{\frac{\tau}{2\pi}}\exp(-i\omega t)\,i\omega\underset{(d)}{\int_0^t \frac{V_g - V_p}{\sqrt{t-t'}}\exp(i\omega t')\,dt'} \qquad [16]$$

The Basset history term must be time independent, its value is evaluated for t rising to infinity. Physically, it is equivalent to suppose that the oscillatory regime is established and to ignore the establishment transient modes. With an appropriate variable change, the Fresnel integrals appear. Finally, the ratio of the complex velocities is given by:

$$\frac{v'_p}{v'_g} = \frac{1 + \frac{3}{2}i\omega\tau + \frac{3}{2}(i+1)\sqrt{\omega\tau}}{1 + (\chi + \frac{1}{2})i\omega\tau + \frac{3}{2}(i+1)\sqrt{\omega\tau}} \qquad [17a]$$

Writing z as the left hand side of this equation and $F(\chi, \omega, \tau)$ as the right hand one, then:

$$z = F(\chi, \omega\tau) = f(\chi, \omega\tau_v)\ (\tau_v = \chi\tau) \qquad [17b]$$

In the analysis, the integrated formula for the Basset force is used. A more simple method may be used for the specific case of a sphere oscillating in translation in a fluid. This method, used by Landau & Lifshitz (1971), leads to the expression of the force applied to the moving particle. It is also necessary to take into consideration the component of the instantaneous pressure gradient (Maxey & Riley 1983; Gatignol 1983; Atkinson & Kytomaa 1992). The complex velocities ratio calculated with this total force, whose expression is valid for any time t, is identical to our result. Thus the assumption of infinite time t, necessary in our treatment for the Basset history calculus, is not restrictive.

When looking at the function F, it can be shown that if $\omega\tau$ is different from zero, then the values of the velocities are different and a phase-lag between those velocities appears. Let us study that function. There are two limiting cases, corresponding to extreme values of $\omega\tau$:

$\omega\tau$ tends to zero

This situation occurs when ω is very small or when the relaxation time decreases. In the first case, the perturbation is very slow. In the second case, the particles get smaller and smaller. In both cases, the particles follow perfectly the carrier fluid velocity, thus the suspension is in an equilibrium state. In the next, the subscript (e) refers to this equilibrium state:

$$\lim_{\omega\tau \to 0} z = z_e \qquad [18]$$

$\omega\tau$ approaches infinity

When ω approaches infinity, the perturbation is so fast that there is not enough time for the particles to react. For large relaxation times, the particles are very big or they may have a very large mass density. This case can also occur when the fluid viscosity is very small since the fluid slides over the particles without being able to drag them well. In all those examples, the particles do not react to the velocities fluctuations of the fluid. Thus, the particles behave like an obstacle placed in the flow. This second limiting case corresponds to a flow that is called "frozen", indicated by the subscript (∞):

$$\lim_{\omega\tau \to \infty} z = z_\infty \qquad [19]$$

The function z is studied in the following section. Starting with only the drag force, then other different terms are added.

(a) Stokes drag. When the term (a) is only present in the second member of [17a], it is obtained:

$$z = \frac{1}{1 + i\chi\omega\tau}, \qquad [20a]$$

That is the equation of a circle, whose radius is 1/2 and centred in ($x = 1/2$, $y = 0$), x and y being respectively the real and imaginary parts of the complex number z. The only half-circle, corresponding to negative values of y, is described in the direct way for increasing values of $\omega\tau$ (figure 1).

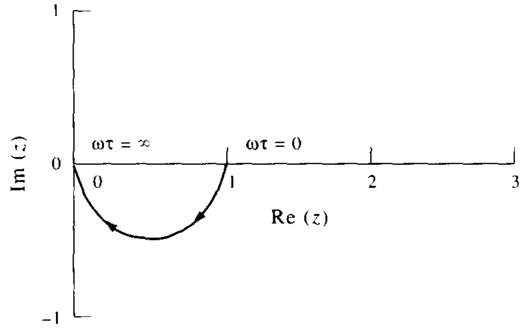

Figure 1. z in the complex plane. Total force = Stokes drag.

$$z_e = 1, \quad z \sim 1 - i\omega\tau\chi \quad \text{for} \quad \omega\tau \ll 1 \tag{20b}$$

$$z_\infty = 0 \tag{20c}$$

(b) Stokes drag and pressure gradient. For this case, the terms (a) and (c) are present, the following result is obtained:

$$z = \frac{1 + i\omega\tau}{1 + i\chi\omega\tau} \tag{21a}$$

The representative curve is the upper-half circle for χ smaller than unity and the lower half-circle for χ larger than unity. For $\chi = 1$, one obtains the point $(x = 1, y = 0)$. The very great importance of the pressure gradient for small values of χ, that is to say for particles which are lighter than the fluid, is observed. For $\chi = 0$, the curve is the vertical upper half-line. For increasing values of χ, the half-circle of figure 1 is found again. Figure 2 shows the function $F(\chi, \omega\tau)$ in the complex plane for the following values of χ: $\chi = 0$, $\chi = 0.25$, $\chi = 2.5$, $\chi \to \infty$. The point $(x = 1, y = 0)$ belongs to all the curves.

$$z_e = 1, \quad z \sim 1 + i\omega\tau(1 - \chi) \quad \text{for} \quad \omega\tau \ll 1 \tag{21b}$$

$$z_\infty = 1/\chi \tag{21c}$$

z_∞ is now χ dependent.

(c) Stokes drag, pressure gradient and virtual mass. The only missing term is the Basset history term (d):

$$z = \frac{1 + \frac{3}{2} i\omega\tau}{1 + (\chi + 1/2)i\omega\tau} \tag{22a}$$

The representative curves are still half-circles passing all by the point $(x = 1, y = 0)$. The curves have been plotted for the previous values of χ (figure 3), they are now in a finite zone. The limiting cases $\chi = 1$, giving the point $(x = 1, y = 0)$ and $\chi \to \infty$, giving the lower half-circle whose radius is equal to 1/2, centred in $(x = 1/2, y = 0)$ are unchanged.

$$z_e = 1, \quad z \sim 1 + i\omega\tau(1 - \chi) \quad \text{for} \quad \omega\tau \ll 1 \tag{22b}$$

$$z_\infty = 3/(2\chi + 1) \tag{22c}$$

(d) The whole terms. The expression of z is ([17a]):

$$z = \frac{1 + \frac{3}{2} i\omega\tau + \frac{3}{2}(i + 1)\sqrt{\omega\tau}}{1 + (\chi + \frac{1}{2})i\omega\tau + \frac{3}{2}(i + 1)\sqrt{\omega\tau}} \tag{23a}$$

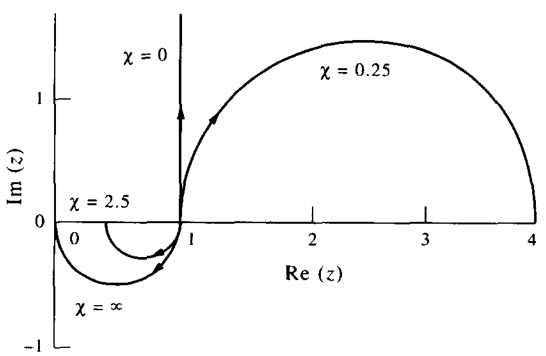

Figure 2. z in the complex plane. Total force = Stokes drag + pressure gradient.

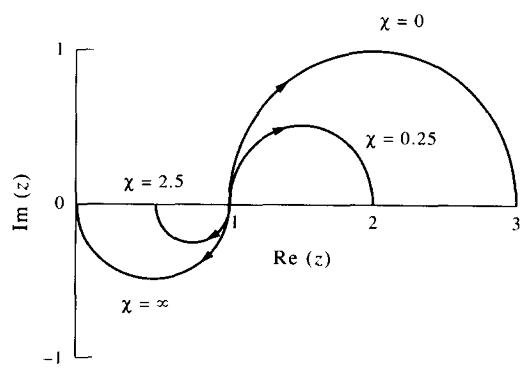

Figure 3. z in the complex plane. Total force = Stokes drag + pressure gradient + virtual mass effect.

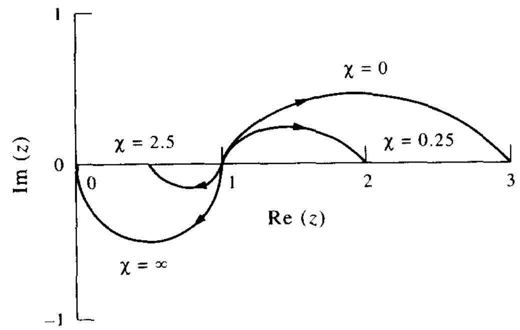

Figure 4. z in the complex plane. Total force = the whole terms.

The curves are no longer circles. The expression of the imaginary part of z shows that its sign is opposite to the sign quantity of $(\chi - 1)$. For χ rising to infinity, y is negative and only the half-circle $y < 0$ is valid. We observe a flatness of the different curves compared to the previous cases (figure 4). The limit curve for $\chi \to \infty$ is identical to the curves of figures 1, 2 and 3.

To study more precisely the Basset term, the z modulus and its argument (phase-lag between v'_p and v'_g) have been plotted. Two cases have been distinguished. In the first one, we take all the forces, in the second one we neglect the Basset force. These curves are plotted as a function of the square root of the reduced angular frequency $\sqrt{\omega\tau}$ (the most little power of $\omega\tau$ in [23a]). From figure 5, showing the ratio of the velocities moduli, one can see that the less heavy particles ($\chi < 1$), moving faster than the fluid, go slower when the Basset force is applied to the particles [figure 5(a)]. That is the inverse on a little zone corresponding to very small values of $\omega\tau$ [figure 5(b)]. The particles, which are heavier than the fluid ($\chi > 1$), move slower than it ($|v'_p/v'_g| < 1$). Now, the Basset term presence makes the particle velocity larger [figure 5(a)], except on a small range of low

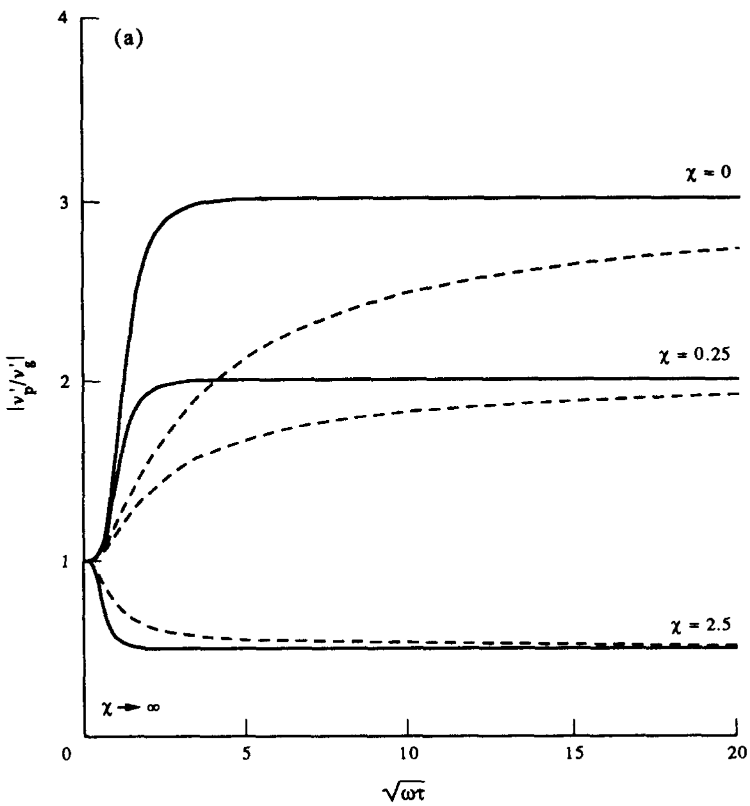

Figure 5(a). $|v'_p/v'_g|$ as a function of $\sqrt{\omega\tau}$. Without the Basset term (——); the whole terms (– – –).

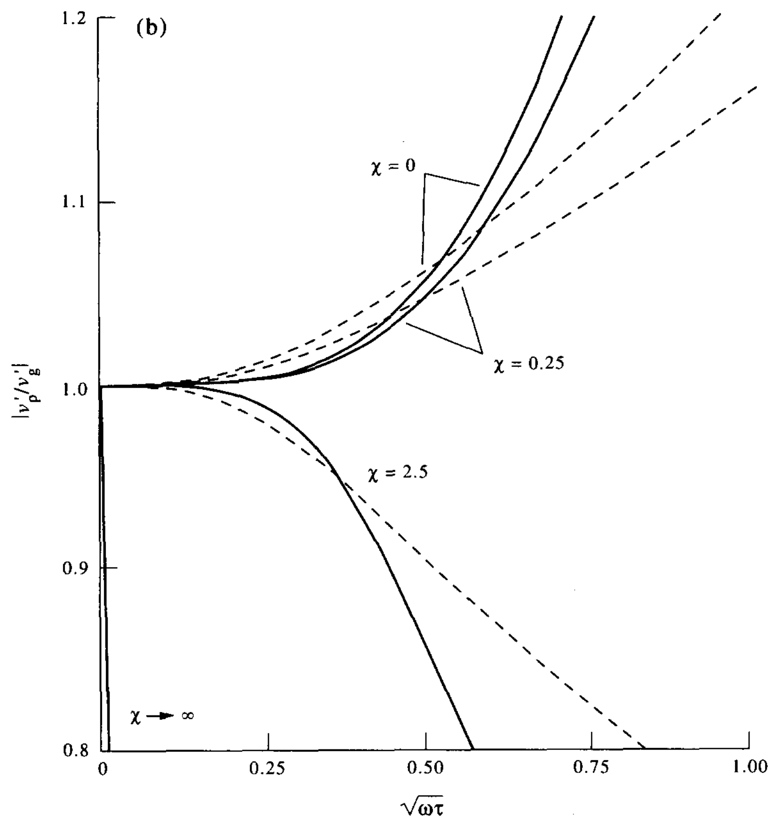

Figure 5(b). $|v_p'/v_g'|$ as a function of $\sqrt{\omega\tau}$ for very small values of $\sqrt{\omega\tau}$. Without the Basset term (——); the whole terms (- - -).

values of $\omega\tau$ [figure 5(b)]. We note on figure 6 that the phase-lag between v_p' and v_g' is positive when the particles are lighter than the fluid and negative in the inverse case. The Basset term first reduces the phase-lag intensity and makes it larger after a certain frequency depending on χ.

$$z_e = 1, \quad z \sim 1 + i\omega\tau(1-\chi) \quad \text{for} \quad \omega\tau \ll 1 \qquad [23b]$$

$$z_\infty = 3/(2\chi + 1) \qquad [23c]$$

The study of the limiting cases shows several things. If $\omega\tau$ tends to zero, then the function F is, as planned, almost equal to unity and $v_p' = v_g'$. We have got the same result (except in case a) for any ω when χ is equal to unity, since the two components have the same mass density.

For large values of $\omega\tau$, the particles move quicker than the fluid if they are lighter than it, slower if they are heavier. That is not the case if the only force applied to the particles is the Stokes drag force since z_∞ is χ independent ($z_\infty = 0$ and $v_p' = 0$).

PART 2. PROPAGATION AND DAMPING OF AN ACOUSTICAL WAVE IN A COMPRESSIBLE TWO-PHASE FLOW

During their propagation, the acoustical waves lose some energy. The causes of these losses are multiple:

1—A geometrical damping, that is the case for the spherical waves.

2—The momentum exchanges associated with the viscosity of the fluid, the exchanges associated to the thermal conductivity.

3—The molecular relaxation due to the non-equilibrium internal energy modes and the chemical relaxation.

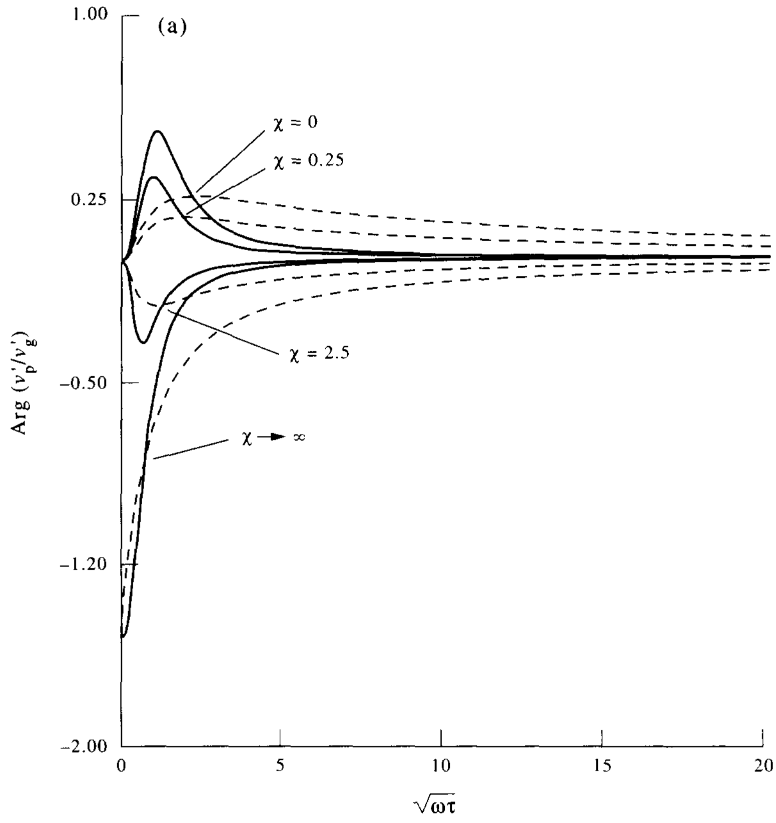

Figure 6(a). Phase-lag between v_p' and v_g' as a function of $\sqrt{\omega\tau}$. Without the Basset term (——); the whole terms (- - -).

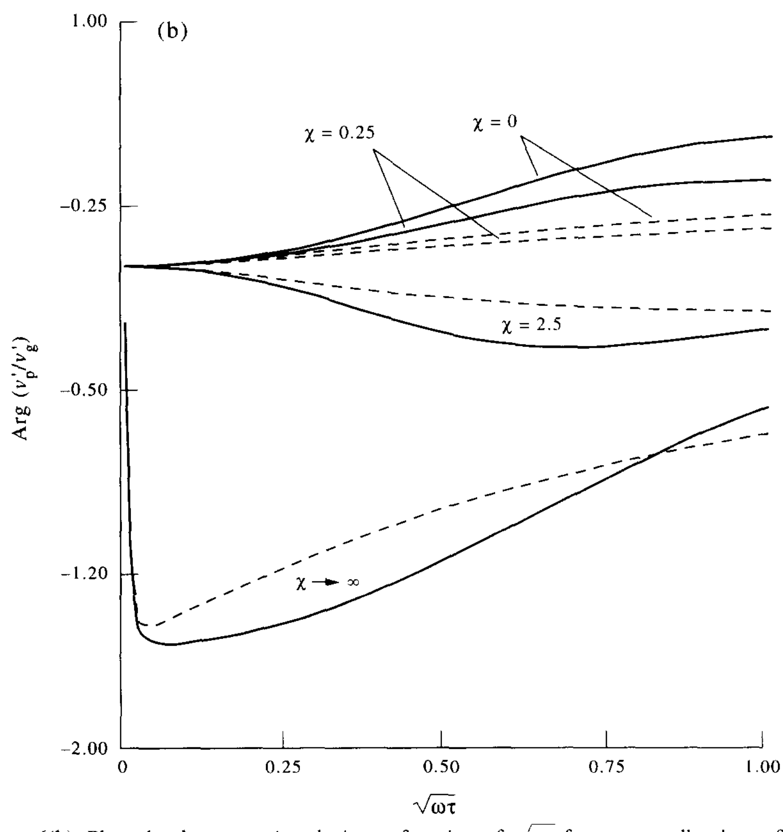

Figure 6(b). Phase-lag between v_p' and v_g' as a function of $\sqrt{\omega\tau}$ for very small values of $\sqrt{\omega\tau}$. Without the Basset term (——); the whole terms (- - -).

If the fluid contains particles, then other sources of damping may appear:

4—The momentum relaxation of the particles. Indeed, the velocity difference between the two phases involves the dissipation of energy at the surface and in the wake of the particle, hence a decrease of the acoustical intensity.

5—The relaxation temperature: some temperature gradients, due to the propagation of the acoustical wave, appear at the surface of the particle. Thus, important losses occur by thermal diffusion.

6—Mass exchanges during evaporation, condensation or combustion of the droplets.

It should be noted that the damping in a turbulent flow is larger than the damping in the laminar flow. Indeed, the acoustical waves are diffused by the velocity and the temperature fluctuations (Candel 1980–1981). The effect of the particles surface tension is small for liquid or solid particles, this effect is no more negligible for bubbles in a liquid. The magnitude of a plane wave decreases with the distance x. The damping law is exponential: $\exp(-bx)$, b is the damping coefficient.

There is no energy loss when an acoustical wave propagates in a one-phase flow, with no molecular or chemical relaxations and where viscous effects and thermoconduction are neglected. Landau & Lifschitz (1971) express the average value of energy dissipation taking into account those two last phenomena. The calculation supposes that the damping is small. In that way, the relative decrease of magnitude is small on a distance about the wave-length: $bc/\omega \ll 1$. In this case, b may be expressed in term of the velocity:

$$b = \frac{\omega^2}{2\rho_{Gs}c^3}\left[\left(\tfrac{4}{3}\mu + \zeta\right) + \varkappa\left(\frac{1}{C_v} - \frac{1}{C_p}\right)\right]$$

(ζ is the gas bulk viscosity, $\varkappa$ the thermal conductivity, C_v and C_p the specific heats of the gas at constant volume and pressure respectively).

These effects are often negligible. For example, in the case of air at a temperature of 288.15 K, we obtain: $b = O(\omega^2\,10^{-13})$. Therefore, this dissipation is not taken into account in the following section, where the assumptions of the first part are maintained. In addition, the following assumptions are made:

—the gaseous phase is a thermally perfect gas (the thermal conductivity is negligible, C_p and C_v are temperature independent).

—the only exchanges between the two phases are momentum and thermal exchanges (no mass transfer). These exchanges occur in the immediate neighborhood of the particles.

The kinetic relaxation time τ_v has been already defined in part one. From now on, we need to introduce a thermal relaxation time to characterize the exchanges between the gas and the particles. These exchanges vanish just in the case of particles whose surface is adiabatic. The thermal relaxation time τ_t is defined as being the ratio between the excess heat and the thermal flux between the particle and the fluid:

$$\tau_t = \frac{m_p C_c (T_G - T_p)}{q_p} \qquad [24]$$

(C_c is the specific heat of the particle, T_G and T_p the absolute temperatures of the gas and of the particle respectively, q_p is the thermal flux between the gas and the particle.)

If τ_t is large, compared to the characteristic time T of the fluid, then the particles temperature is nearly independent on the fluid temperature, that is the case of inert particles. If τ_t is much smaller than T then the particles temperature is strongly dependent on the fluid temperature.

The radiative thermal exchanges are neglected to derive the expression of τ_t. It is admitted that the heat flux q_{ps} between the particle and its surface, to the temperature T_s, and the flux between that surface and the gas q_{sG} may be written as: $q_{ps} = a_p(T_p - T_s)$ and $q_{sG} = a_G(T_s - T_G)$. Since there is no condensation or evaporation, we are allowed to write: $q_{ps} = q_{sG} = q_p$. If the particle is not thermally homogeneous, Markatos (1986) gives an expression for the coefficient a_p, proportional to the thermal conductivity of the particles. In this paper, we suppose that the particles temperature is uniform ($T_p = T_s$). It is equivalent to say that the particles are very small or that their thermal diffusivity rises to infinity. We obtain the following result:

$$q_p = Sh_c(T_G - T_p), \quad [25]$$

where S is the particle surface area and h_c is the convection coefficient.

The determination of the thermal exchange coefficient h_c sets the same problem that we have met for calculating the Stokes drag force in the momentum equation. The transitory effects are neglected and just the local effects of the flow around the particle are taken into account. The Nusselt number is:

$$\text{Nu} = h_c\, d/\varkappa \quad [26]$$

The Stokes assumption corresponds to a pure conduction transfer. For the case of a sphere, that leads to the following result: Nu = 2. That is a reasonable assumption since the Reynolds number is very small. Consequently:

$$q_p = (T_G - T_p)\, d\varkappa 2\pi$$

$$\frac{q_p}{m_p} = \frac{(T_G - T_p)3\varkappa}{\rho_{ps} r_a^2}, \quad [27]$$

where ρ_{ps} is the specific density of the particles.

Finally:

$$\tau_t = \frac{C_c r_a^2 \rho_{ps}}{3\varkappa} \quad [28]$$

The introduction of correlation factors (Mach number etc.) allows to take into account the compressibility effects of the flow. As for τ_v, it is not necessary in the framework of this analysis since the perturbations are very small. The thermal relaxation time may be written as a function of τ_v:

$$\tau_t = \tfrac{3}{2}\,\text{Pr}\,\beta\tau_v, \quad [29]$$

where Pr is the Prandtl number and β is the ratio of the specific heats of the two phases: C_c/C_p.

Thus, in this particular case, the characteristic times have no independent values.

Let us write the continuity, the momentum and the energy equations for each phase (two fluids model):

$$\frac{\partial \rho_G}{\partial t} + \text{div}(\rho_G \mathbf{v}_G) = 0 \quad [30]$$

(continuity equation of the gas)

$$\frac{\partial \rho_p}{\partial t} + \text{div}(\rho_p \mathbf{v}_p) = 0 \quad [31]$$

(continuity equation of the particles)

In these two equations, the mass densities are given by:

$$\rho_G = \rho_{Gs}\epsilon \quad [32a]$$

$$\rho_p = \rho_{ps}(1 - \epsilon) \quad [32b]$$

ϵ is the volume fraction occupied by the gaseous phase.

$$\rho_G \frac{\partial \mathbf{v}_G}{\partial t} + \rho_G \mathbf{v}_G \cdot \overrightarrow{\text{grad}}\,(\mathbf{v}_G) + \overrightarrow{\text{grad}}\,(p) + \rho_p \frac{\partial \mathbf{v}_p}{\partial t} + \rho_p \mathbf{v}_p \cdot \overrightarrow{\text{grad}}\,(\mathbf{v}_p) = \mathbf{0} \quad [33]$$

(momentum equation of the mixture)

The momentum equation of the dispersed phase has been studied in the first part ([3]). The body forces are ignored.

e being the internal energy per unit mass, we have:

$$\rho_G \frac{\partial (e_G + v_G^2/2)}{\partial t} + \rho_G \mathbf{v}_G \cdot \overrightarrow{\text{grad}}\,(e_G + v_G^2/2) + \text{div}(p\mathbf{v}_G) + \rho_p \frac{\partial (e_p + v_p^2/2)}{\partial t} + \rho_p \mathbf{v}_p \cdot \overrightarrow{\text{grad}}\,(e_p + v_p^2/2) = 0 \quad [34]$$

(conservation of energy of the mixture)

$$\frac{\partial T_p}{\partial t} + \mathbf{v}_p \cdot \overrightarrow{\text{grad}}\, T_p = \frac{T_G - T_p}{\tau_t} \quad [35]$$

(internal energy balance of the dispersed phase)

$$p = \rho_G r T_G = \rho_{Gs} \varepsilon r T_G \quad [36]$$

(state equation of the perfect gas of constant $r = R/M$, R is the universal gas constant: $R = 8.3144$ J/K/mol and M is the gram molecular weight of the gas)

We take for r the value usually used for one-phase flow.

The steady state is characterized by the phases equilibrium: the velocity is equal to zero and the temperature is uniform. The linearization is carried out by writing:

$$\rho_G = \rho_{Go} + \rho'_G \qquad \rho_p = \rho_{po} + \rho'_p$$
$$T_G = T_{Go} + T'_G \qquad T_p = T_{po} + T'_p$$
$$e_G = e_{Go} + e'_G \qquad e_p = e_{po} + e'_p$$
$$\mathbf{v}_G = \mathbf{v}_{Go} + \mathbf{v}'_G \qquad \mathbf{v}_p = \mathbf{v}_{po} + \mathbf{v}'_p$$
$$p = p_o + p' \quad [37]$$

With:

$$T_{Go} = T_{po} = T_o \quad [38a]$$

$$\mathbf{v}_{Go} = \mathbf{v}_{po} = \mathbf{0} \quad [38b]$$

Around T_o, e'_G and e'_p may be written:

$$e'_G = C_v T'_G \quad [39a]$$

$$e'_p = C_c T'_p \quad [39b]$$

Neglecting the second order terms, we obtain a seven equation linear system with the seven unknown variables ρ'_G, ρ'_p, T'_G, T'_p, $\mathbf{v}'_G$, $\mathbf{v}'_p$, p'.

$$\frac{\partial \rho'_G}{\partial t} + \rho_{Go}\,\text{div}(\mathbf{v}'_G) = 0 \quad [40a]$$

$$\frac{\partial \rho'_p}{\partial t} + \rho_{po}\,\text{div}(\mathbf{v}'_p) = 0 \quad [40b]$$

(continuity equations)

$$\rho_{Go} \frac{\partial \mathbf{v}'_G}{\partial t} + \overrightarrow{\text{grad}}\,(p') + \rho_{po} \frac{\partial \mathbf{v}'_p}{\partial t} = \mathbf{0} \quad [41]$$

(momentum equation)

$$\rho_{Go} \frac{\partial e'_G}{\partial t} + \rho_{po} \frac{\partial e'_p}{\partial t} + p_o\,\text{div}(\mathbf{v}'_G) = 0 \quad [42a]$$

$$\frac{\partial T'_p}{\partial t} = \frac{T'_G - T'_p}{\tau_t} \quad [42b]$$

(energy balance)

$$p' = r(\rho_{Go} T'_G + \rho'_G T_{Go}) \quad [43]$$

(state equation)

We have to add [14] to this system. The entropy balance of the medium allows us to show that the perturbation propagates isentropically in the limit of the first order approximation.

It is not easy to obtain a single equation with only one variable since the expression of the Basset history term is complicated. The final result is a two equation system with the respective unknowns: $\mathbf{v}'_G$ and $\mathbf{v}'_P$:

$$\frac{\partial^3 \mathbf{v}'_G}{\partial t^3} + X\frac{\partial^3 \mathbf{v}'_p}{\partial t^3} - c_0^2\left(\Delta\left(\frac{\partial \mathbf{v}'_G}{\partial t}\right) + \overrightarrow{\mathrm{rot}}\left(\overrightarrow{\mathrm{rot}}\left(\frac{\partial \mathbf{v}'_G}{\partial t}\right)\right)\right)$$
$$- \frac{c_0^2}{\tau_t}(X\beta + 1)(\Delta(\mathbf{v}'_G) + \overrightarrow{\mathrm{rot}}\,(\overrightarrow{\mathrm{rot}}\,(\mathbf{v}'_G))) + \frac{(1 + X\beta\gamma)}{\tau_t}\left(\frac{\partial^2 \mathbf{v}'_G}{\partial t^2} + X\frac{\partial^2 \mathbf{v}'_p}{\partial t^2}\right) = \mathbf{0} \quad [44]$$

Δ being the Laplacian, $\overrightarrow{\mathrm{rot}}$ the rotational. X is equal to the ratio ρ_{po}/ρ_{Go}.

Equation [14] is unchanged. The speed of sound c_o in the gas alone is: $c_o = \sqrt{\gamma r T_o}$, γ is the specific heat ratio of gas: C_p/C_v.

For plane acoustical waves, the solutions where the quantities V_G and V_p of [15a] and [15b] are periodic and space-dependent functions are considered:

$$\mathbf{v}'_G = v'_G \mathbf{i} = v_G \exp[i(\omega t - \mathbf{K}\cdot\mathbf{r})]\mathbf{i} \quad [45a]$$

$$\mathbf{v}'_p = v'_p \mathbf{i} = v_p \exp[i(\omega t - \mathbf{K}\cdot\mathbf{r})]\mathbf{i} \quad [45b]$$

ω is the angular frequency, $\mathbf{K}$ the wave-vector and $\mathbf{r}$ the position vector, v_G, v_p are independent of the variables t and $\mathbf{r}$, $\mathbf{i}$ is the direction of propagation of the acoustical wave.

After projecting the two previous equations along the propagation direction of the wave, we obtain the dispersion equation:

$$\frac{c_0^2 K^2}{\omega^2} = \frac{1 + X\beta\gamma + i\omega\tau_t}{1 + X\beta + i\omega\tau_t}(1 + Xz) \quad [46]$$

where z is given by [17a]:

$$\frac{v'_p}{v'_G} = z = F(\chi, \omega\tau) = f(\chi, \omega\tau_v) \quad [47]$$

In this section, χ is larger than 1 (liquid or solid particles in a gas). The influence on the thermal transfer decreases when: $\gamma \to 1$, $\beta \to 0$, $X \to 0$.

Let us express the complex wave-number K as a function of the angular frequency, the sound velocity and damping:

$$K = \frac{\omega}{c} - ib \quad [48]$$

Thus:

$$\frac{1}{c^2} - \frac{b^2}{\omega^2} = \frac{1}{c_0^2}\mathrm{Re}\left[\frac{1 + X\beta\gamma + i\omega\tau_t}{1 + X\beta + i\omega\tau_t}(1 + Xz)\right] \quad [49a]$$

$$\frac{2b}{\omega c} = \frac{-1}{c_0^2}\mathrm{Im}\left[\frac{1 + X\beta\gamma + i\omega\tau_t}{1 + X\beta + i\omega\tau_t}(1 + Xz)\right] \quad [49b]$$

Re being the real part of the expression, Im its imaginary part.

If X becomes very small, then the particles concentration decreases ($\epsilon \to 1$). The previous formulae give us effectively the characteristic values of a one-phase flow (the wave velocity is equal to c_o and there is no damping).

First, let us study the dispersion equation in limiting cases. As mentioned above, τ_v and τ_t are each dependent ([29]).

$\omega\tau_v$ and $\omega\tau_t$ tend to zero

The particles follow perfectly the fluid velocity and the fluid temperature perturbations. As mentioned before, the suspension is in an equilibrium state:

$$\lim_{\substack{\omega\tau_v \to 0 \\ \omega\tau_t \to 0}} z = z_e \quad [50]$$

The bounded expansions of z and of the dispersion equation give the following results:

(a) Stokes drag. $z \sim 1 - i\omega\tau_v$

$$c_e = c_o \sqrt{\frac{1 + X\beta}{(1 + X\beta\gamma)(1 + X)}} \quad [51]$$

$$b \sim \frac{\omega^2}{2c_o}\tau_v X \sqrt{\frac{(1 + X)(1 + X\beta\gamma)}{(1 + X\beta)}}\left(\frac{1}{1 + X} + \frac{3\,\mathrm{Pr}\,\beta^2(\gamma - 1)}{2(1 + X\beta)(1 + X\beta\gamma)}\right) \quad [52]$$

(b) Stokes drag and pressure gradient. $z \sim 1 + i\omega\tau_v(1 - \chi)/\chi$. The relation between c_e and c_o is identical to the previous case.

$$b \sim \frac{\omega^2}{2c_o}\tau_v X \sqrt{\frac{(1 + X)(1 + X\beta\gamma)}{(1 + X\beta)}}\left(\frac{1 - 1/\chi}{1 + X} + \frac{3\,\mathrm{Pr}\,\beta^2(\gamma - 1)}{2(1 + X\beta)(1 + X\beta\gamma)}\right) \quad [53]$$

(c) Stokes drag, pressure gradient and virtual mass effect. $z \sim 1 + i\omega\tau_v(1 - \chi)/\chi$. The same expressions as in the case b are found.

(d) The whole terms. $z \sim 1 + i\omega\tau_v(1 - \chi)/\chi$. We find again the same expressions for the velocity and the damping. Thus in every case, we arrive at:

$$c_e = c_o \sqrt{\frac{1 + X\beta}{(1 + X\beta\gamma)(1 + X)}} \quad [54]$$

This velocity is frequency, fluid viscosity and particle size independent. Its value is also smaller than the velocity in the gas alone since $\gamma > 1$.

Furthermore:

$$b \propto \frac{\omega^2\tau_v}{c_o} \propto \frac{\omega^2\rho_{ps}r_a^2}{c_o\mu} \quad [55]$$

For this limiting case, the damping coefficient is small since there is no relative motion between the fluid and the particles. The Biot theory also predicts a damping proportional to ω^2 at low frequencies. Our results agree with Atkinson & Kytomaa (1992) since they obtain: $b \propto \omega^2 r_a^2/\mu$. Thus, at low frequencies, the damping is inversely proportional to the temperature and to the gas velocity (in this last case, the Prandtl number is supposed to be constant). For fixed values of χ and X, the damping increases with increasing ρ_{ps}. If X is very small, we obtain: $b \propto X$. But:

$$X \propto (1 - \epsilon)/\epsilon \simeq 1 - \epsilon, \quad \text{hence:} \quad b \propto 1 - \epsilon \quad [56]$$

The damping is proportional to the particles concentration when this is very small. Gibson & Toksoz (1989) and Allegra & Hawley (1972) observe the same behaviour.

$\omega\tau_v$ and $\omega\tau_t$ approach infinity

We have previously seen the meaning of $\omega\tau_v$ rising to infinity. For very large values of τ_t, the particles are thermally inert and do not react to the temperature fluctuations of the gas. We write:

$$\lim_{\substack{\omega\tau_v \to \infty \\ \omega\tau_t \to \infty}} z = z_\infty \quad [57]$$

(a) Stokes drag. $z_\infty = 0$

The high frequency limit for the sound speed is given by:

$$c_\infty = c_o \quad [58]$$

For the damping factor, we find:

$$b_\infty = \frac{X}{c_o\tau_v}\left(\frac{1}{2} + \frac{\gamma - 1}{3\,\mathrm{Pr}}\right) \quad [59]$$

(b) Stokes drag and pressure gradient. $z_\infty = 1/\chi$

$$c_\infty = c_0/\sqrt{(1 + X/\chi)} < c_o \quad [60]$$

$$b_\infty = \frac{X}{c_o\tau_v}\sqrt{1 + \frac{X}{\chi}}\left(\frac{1}{2}\frac{\chi - 1}{\chi + X} + \frac{\gamma - 1}{3\,\mathrm{Pr}}\right) \quad [61]$$

Now, the wave velocity is smaller than c_o and it depends on X/χ.

(c) Stokes drag, pressure gradient and virtual mass effect. $z_\infty = 3/(2\chi + 1)$

$$c_\infty = c_o\sqrt{\frac{2\chi + 1}{3X + 1 + 2\chi}} < c_o \quad [62]$$

$$b_\infty = \frac{X}{c_o\tau_v}\sqrt{\frac{1 + 2\chi + 3X}{1 + 2\chi}}\left(\frac{2\chi(\chi - 1)}{(2\chi + 1)(1 + 2\chi + 3X)} + \frac{\gamma - 1}{3\,\mathrm{Pr}}\right) \quad [63]$$

(d) The whole terms. $z_\infty = 3/(2\chi + 1)$

$$c \sim c_o\sqrt{\frac{2\chi + 1}{3X + 1 + 2\chi}}\left(1 + \frac{3\sqrt{\chi}(\chi - 1)X}{\sqrt{\omega\tau_v}(2\chi + 1)(1 + 2\chi + 3X)}\right)^{-1} \quad [64]$$

$$b \sim \frac{X\sqrt{\omega\tau_v}}{c_o\tau_v}\sqrt{\frac{1 + 2\chi + 3X}{1 + 2\chi}}\left(\frac{3\sqrt{\chi}(\chi - 1)}{(2\chi + 1)(1 + 2\chi + 3X)}\right) \quad [65]$$

We state that in any case the velocity c_∞ is almost equal to c_o. Indeed, it must be kept in mind that, for large values of the relaxation times, the particles behave like an obstacle placed in the flow. That is why the sound speed is nearly equal to the velocity in a one-phase flow.

The damping is found to be proportional to $1/c_o\tau_v$ when the Basset force is omitted. If this force is taken into account, we have the following result:

$$b \propto \left(\frac{\mu\omega}{c_o^2 r_a^2 \rho_{ps}}\right)^{1/2} \quad [66]$$

The Basset history term depends on the square root of the angular frequency, hence the proportionality between the damping and this variable. This result is consistent with the Biot theory. Note that Atkinson & Kytomaa (1992) find the same tendencies since they show that $b \propto (\mu\omega/r_a^2)^{1/2}$. Thus, the behaviours for large and small frequencies are very different. Furthermore, if the particles concentration is very small, this leads to the following linear dependence:

$$b \propto 1 - \epsilon \quad [67]$$

GENERAL CASE

Now, let us study the wave behaviour for any values of $\omega\tau_v$ and $\omega\tau_t$. Let us take the example of water droplets in air (fog) with: $T_{Go} = 288.15$ K and $p_o = 1.013 \times 10^5$ Pa. The physical characteristics of the gaseous phase are: $C_p = 1012$ J/kg/K, $\gamma = 1.401$, $\rho_{Gs} = 1.225$ kg/m^3,

$\mu = 1.78 \times 10^{-5}$ kg/m s, $\varkappa = 2.51 \times 10^{-2}$ J/ms K and $M = 29$ g/mol. For the dispersed phase, we have: $C_c = 4.1858 \times 10^3$ J/kg/K, $\rho_{ps} = 999.099$ kg/m^3. Thus, it comes the following values:

$$\chi = 815.59 \qquad c_o = 340.087 \text{ m/s}$$

$$\nu = 1.453 \times 10^{-5}\, m^2/s \qquad r = 286.703 \text{ J/kg/K}$$

$$\text{Pr} = 0.7176 \qquad \beta = 4.1361$$

$$\tau_v = 1.2473 \times 10^{-3} \text{ s} \qquad \tau_t = 5.554 \times 10^{-3} \text{ s} \quad \text{thus:} \quad \tau_v/\tau_t = 0.225$$

Let us see the different parameters influence on the velocity and on the damping coefficient. The continuous curves correspond to the case where only the Stokes drag force is taken into account (case number one) and the dashed lines are valid when the whole forces act on the particles (case number two). The obtained curves when the virtual mass effect or the pressure gradient are taken into account are almost the same as in case number one.

Influence of the product $\omega\tau_v$ ($r_a = 10^{-5}$ m and $1 - \epsilon = 5.0 \times 10^{-4}$)

We plot three types of curves:

—the ratio: c/c_e as a function of $\log_{10}(\omega\tau_v)$.
—the damping coefficient b as a function of $\log_{10}(\omega/\omega_o)$, $\omega_o = 1$.
—the reduced damping coefficient per unit wave-length: $B = 2\pi bc_e^2/\omega c$ as a function of $\log_{10}(\omega\tau_v)$.

The curve [figure 7(a)] shows that the velocity increases with increasing values of $\omega\tau_v$, furthermore: $c_e < c < c_\infty < c_o$. If the Basset force acts on the particle, then the acoustical wave velocity decreases. In the first case, $c_e = 256.23$ m/s, $c_\infty = 340.09$ m/s, thus: $c_\infty/c_e = 1.33$. Since, the term X/χ is very small, we have $c_\infty \simeq c_o$. We recall here that the equilibrium velocity does not depend on the studied combination of the forces acting on the suspension.

The reduced damping [figure 7(b)] first increases, reaches a maximum value B_m for ω_m, and then decreases. We have: $\log_{10}(\omega_m\tau_v) < 0$, so $\omega_m < 1/\tau_v$. In the first case $B_m = 0.66$, in the second case, that value is smaller since $B_m = 0.64$. Besides a certain value of $\omega\tau_v$, the reduced damping calculated when all the forces apply to the particles is larger than the one obtained when just the Stokes drag force is taken into account.

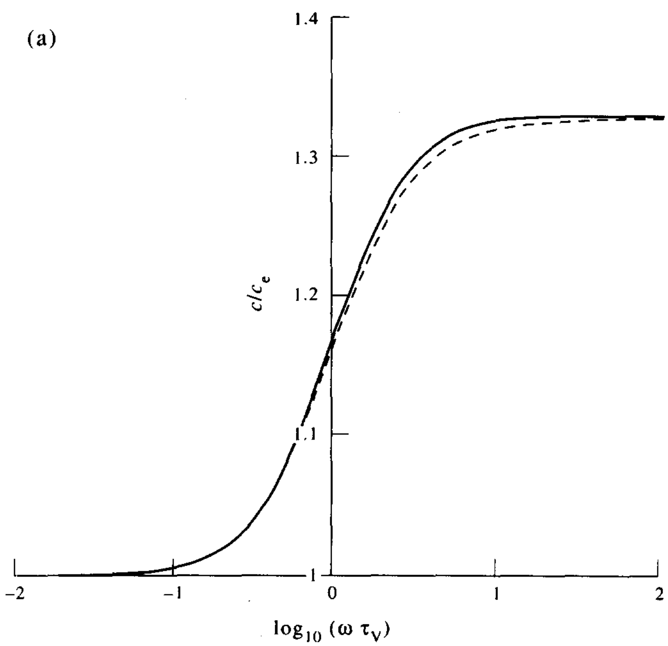

Figure 7(a). c/c_e as a function of $\log_{10}(\omega\tau_v)$. Stokes drag (——); the whole terms (– – –).

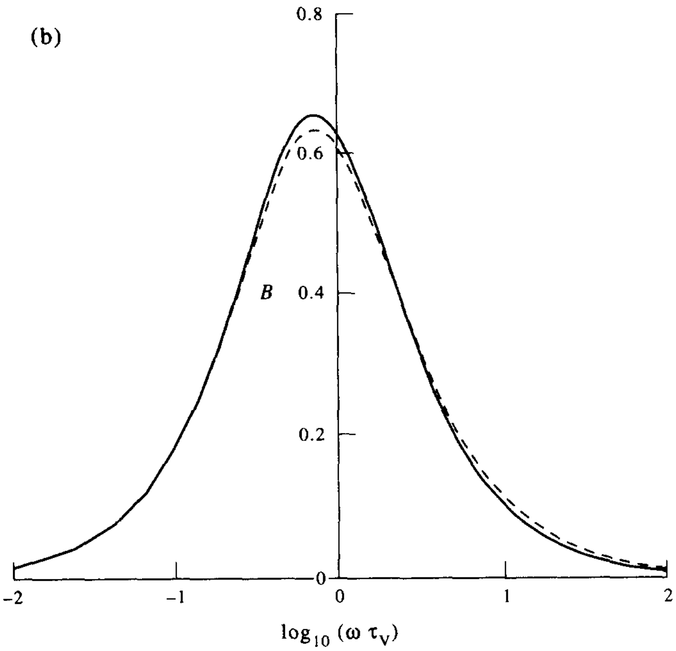

Figure 7(b). B as a function of $\log_{10}(\omega\tau_v)$. Stokes drag (——); the whole terms (- - -).

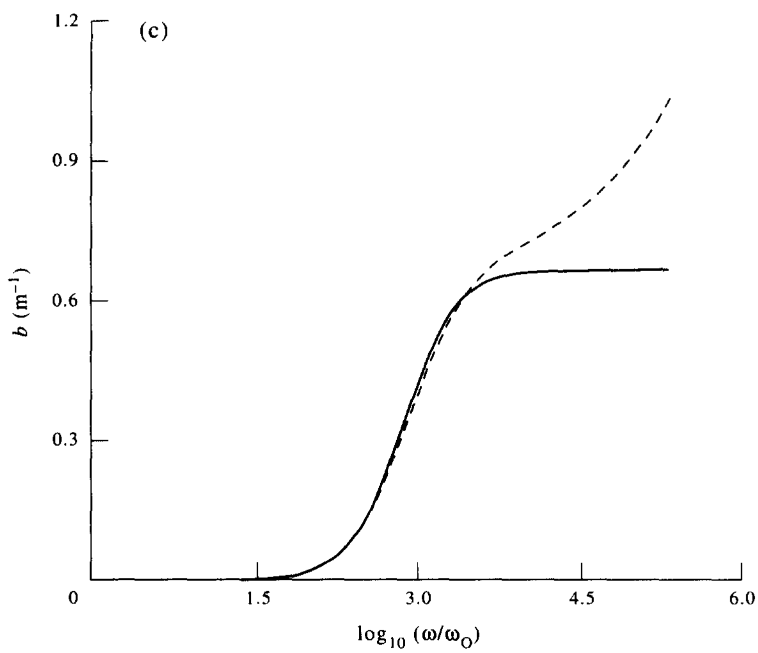

Figure 7(c). b (m^{-1}) as a function of $\log_{10}(\omega/\omega_0)$. Stokes drag (——); the whole terms (- - -).

The figure 7(c) shows the damping b as a function of $\log_{10}(\omega/\omega_0)$. Besides a certain frequency, the damping calculated in the second case is larger than the one obtained in case number one. It reaches a limit value when the Basset term is overlooked but it tends to infinity in the inverse case ($b \sim \sqrt{\omega}$). This different behaviour does not appear when the reduced damping is plotted.

Influence of the particles concentration ($r_a = 10^{-5}$ m and ω = 1000 rad/s)

The velocity decreases with $1 - \epsilon$ [figure 7(d)]. The damping increases for increasing values of the particles concentration [figure 7(e)]. We remark that the velocity is smaller when the Basset

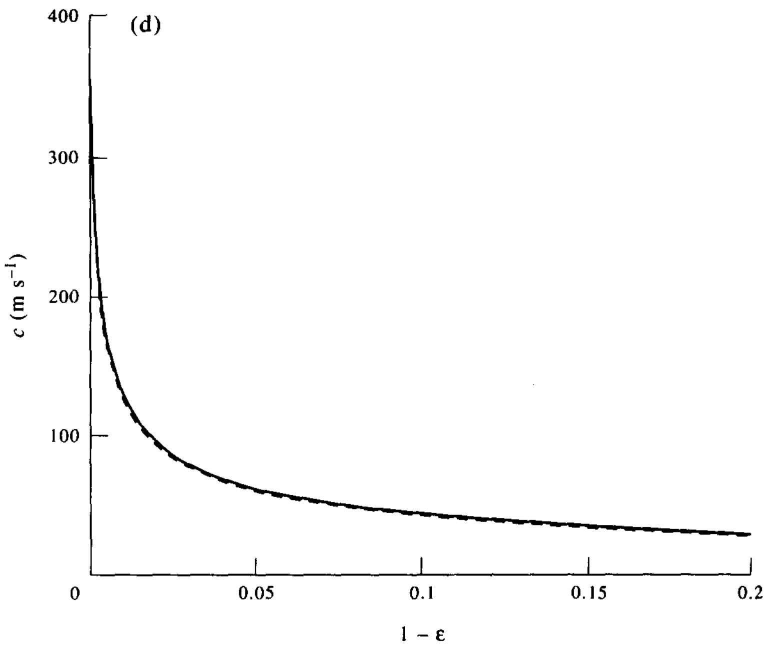

Figure 7(d). c (m s^{-1}) as a function of $1-\epsilon$. Stokes drag (——); the whole terms (---).

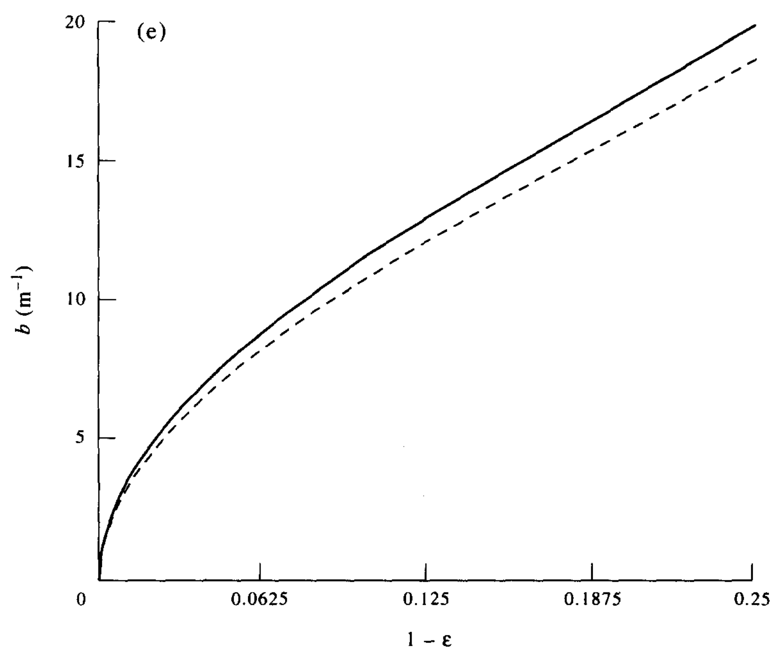

Figure 7(e). b (m^{-1}) as a function of $1-\epsilon$. Stokes drag (——); the whole terms (---).

term is taken into consideration. For this given angular frequency, the Basset term reduces the damping. Those last results are predictable from figure 7.

Influence of particles radius ($1-\epsilon = 5 \times 10^{-4}$ and $\omega = 1000$ rad/s)

As was expected from the study of the velocity as a function of $\omega\tau_v$, the velocity increases from c_c to c_∞ and the Basset term involves a decrease of its value [figure 7(f)]. For the damping [figure 7(g)], we observe a non-monotonic behaviour. The damping first increases quickly and then decreases

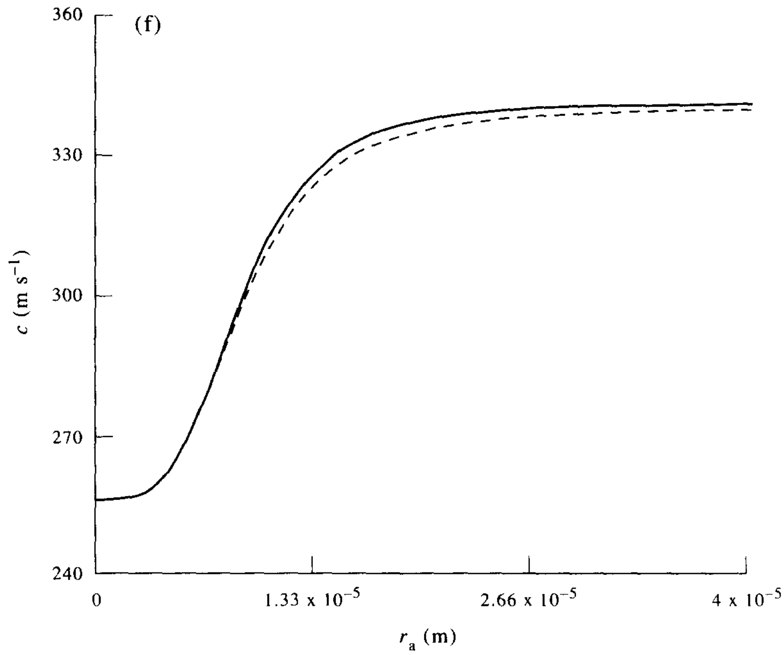

Figure 7(f). c (m s^{-1}) as a function of r_a (m). Stokes drag (——); the whole terms (- - -).

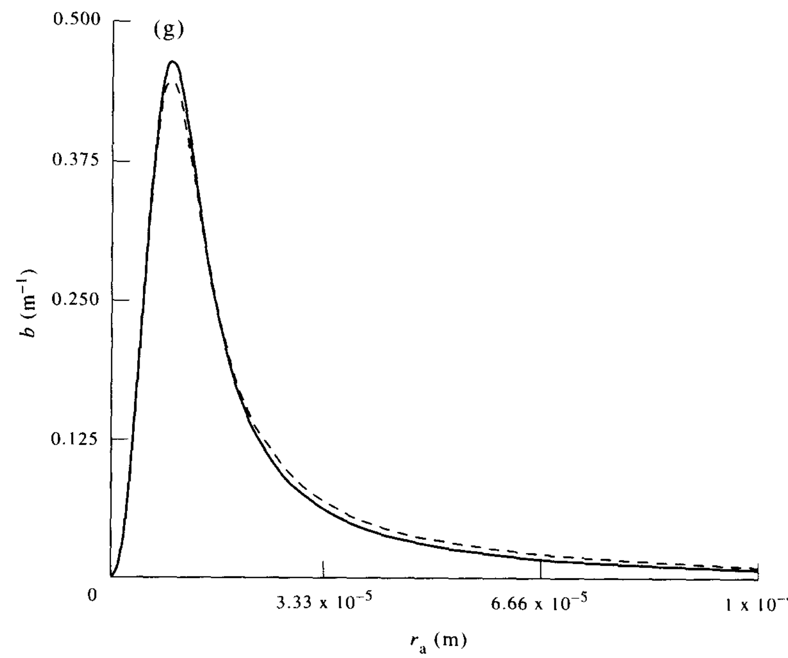

Figure 7(g). b (m^{-1}) as a function of r_a (m). Stokes drag (——); the whole terms (- - -).

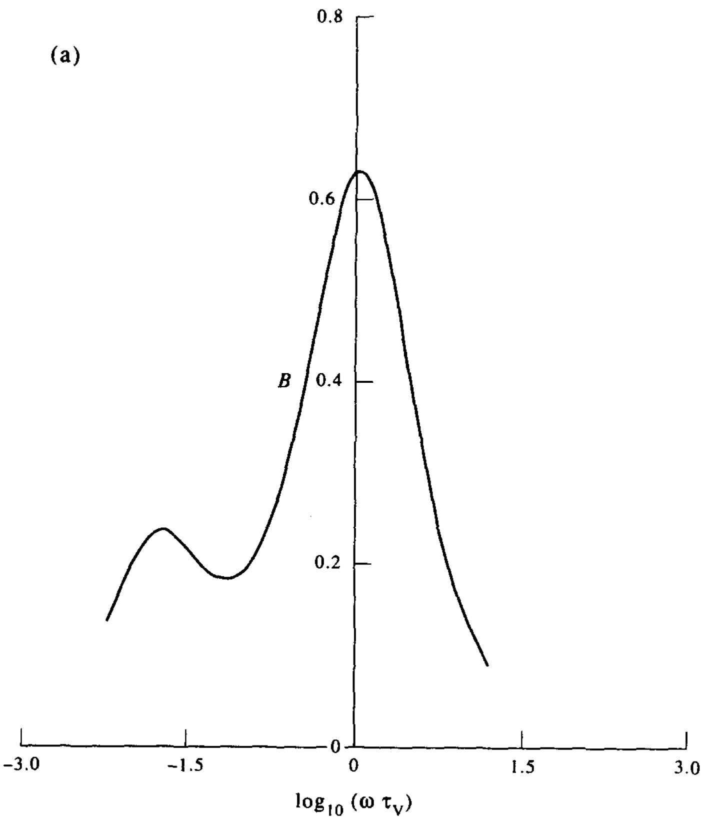

Figure 8(a). Thermally inert particles: $\tau_t/\tau_v = 100$, B as a function of $\log_{10}(\omega\tau_v)$.

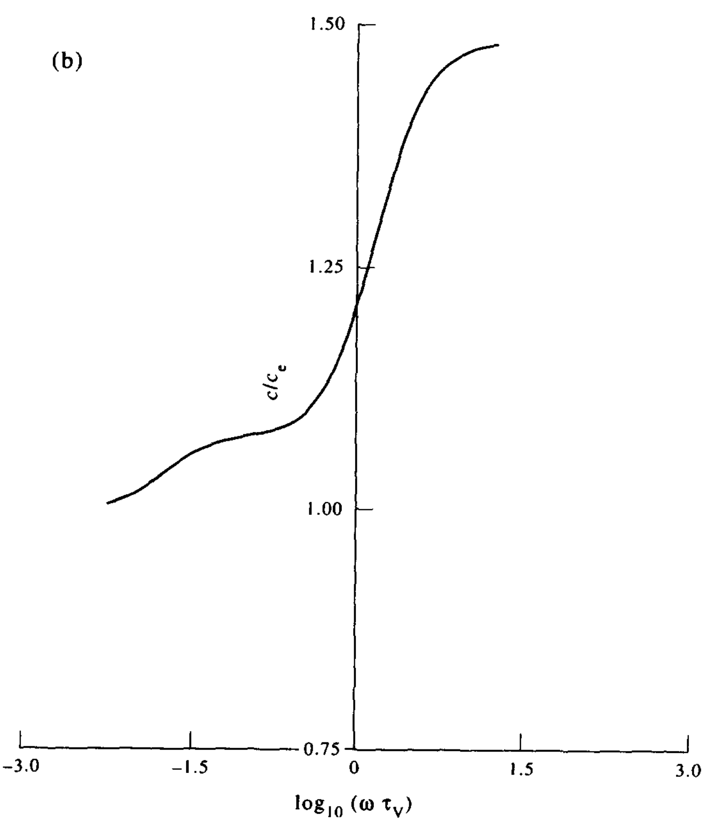

Figure 8(b). Thermally inert particles: $\tau_t/\tau_v = 100$, c/c_e as a function of $\log_{10}(\omega\tau_v)$.

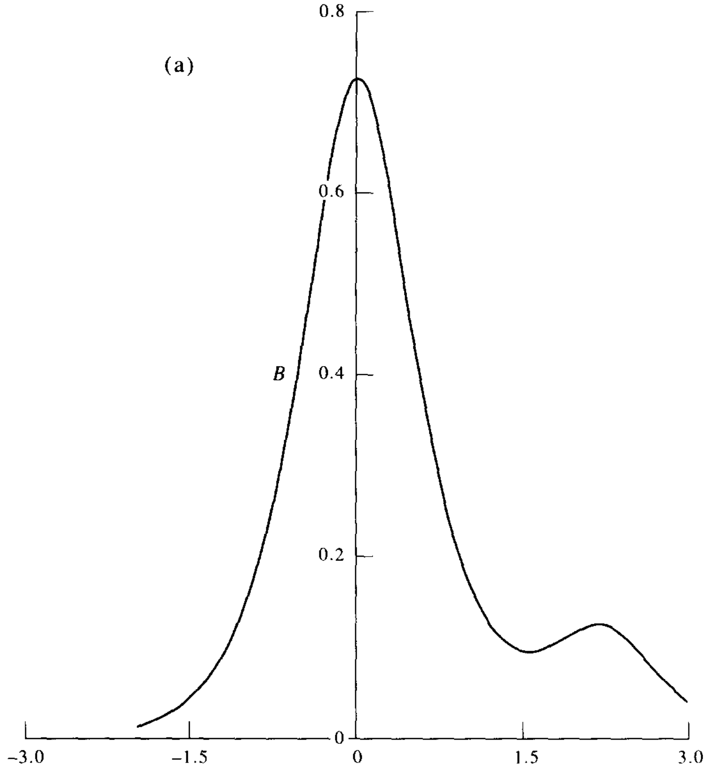

Figure 9(a). Particles in a rarefied gas or easily deformable particles: $\tau_t/\tau_v = 1/100$, B as a function of $\log_{10}(\omega\tau_v)$.

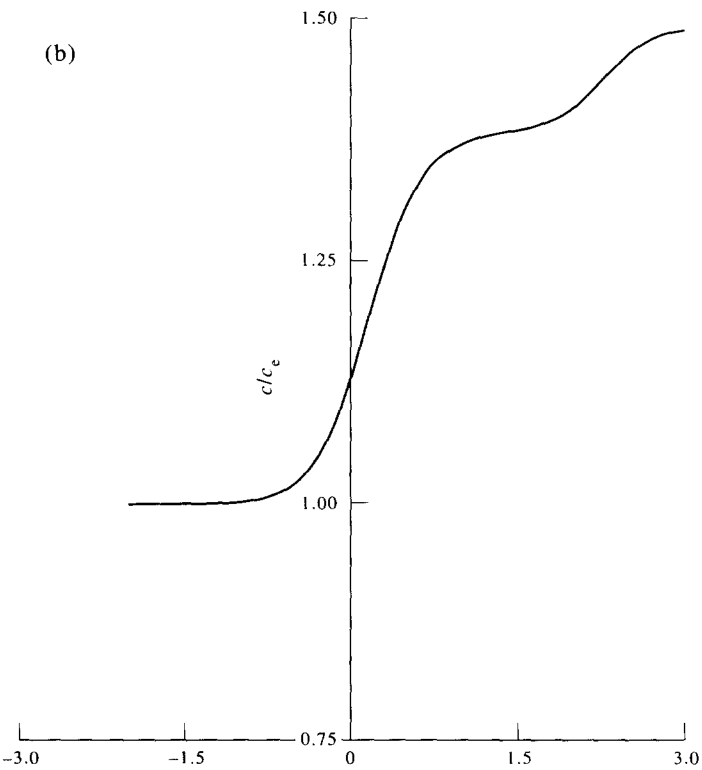

Figure 9(b). Particles in a rarefied gas or easily deformable particles: $\tau_t/\tau_v = 1/100$, c/c_e as a function of $\log_{10}(\omega\tau_v)$.

after having reached a maximum in $r = r_m$. It is shown that:

$$r_m \simeq \left(\frac{9\mu}{2\omega\rho_{ps}}\right)^{1/2} \qquad [68]$$

Urick (1948) gives an explanation for this phenomenon. He shows that the damping coefficient is proportional to the celerities ratio:

$$b \,\alpha\, \frac{\overline{(v_G - v_p)^2}}{\overline{v_G^2}} \qquad [69]$$

The sign "——" indicates the mean temporal value of the concerned quantity. If the particles grow bigger, they move more difficultly and $(v_G - v_p)^2$ increases since v_p decreases. Thus, the damping increases. But when the radius increases, the total surface of the particles decreases making the damping smaller (the particles concentration being kept constant).

For small values of the radius, the damping in case number one is larger than the one calculated in case number two, this behaviour reverses when the radius increases.

Influence of the ratio τ_v/τ_t

To obtain [29], we suppose that the particles are very small or that their thermal diffusivity is infinite. For gases, the Prandtl number is nearly equal to unity. Furthermore, in the most usual cases, the β order of magnitude is also one. Then τ_v and τ_t have approximately the same value: $\tau_v/\tau_t = O(1)$. If the thermal diffusivity of the particle is small or if the particle is big, [29] is no longer valid since the temperature is no longer uniform in the sphere. For a particle which is very thermally inert, we may write: $\tau_v/\tau_t \ll 1$.

The inverse case: $\tau_v/\tau_t \gg 1$, is more difficult to realize. Indeed, the case of a particle whose thermal diffusivity is infinite has been studied, so we must act on τ_v. We should find a kinetic relaxation time which is larger than the one expected in the Stokes theory. It must be kept in mind that this theory is only applicable for very small particles' Reynolds numbers, the particles being spherical, rigid and perfectly smooth. If the roughness of the particle is large, then it follows the fluid motion easily. On the other hand, if the particle is made of a material which is not very dense or not very viscous, then internal movements occur inside the particle. This transfer momentum is not used to carry the particle away, thus τ_v increases. This example corresponds to easily deformable particles. Particles moving in a rarefied gas also have a large kinetic relaxation time since the fluid slips on the particle surface. In that way, the spheres have difficulty in following the fluid motion. Acting on the particle radius to obtain really different relaxation times is more delicate since τ_v and τ_t are both proportional to the radius. Let us study two cases taking different values for the ratio of the relaxation times.

Thermally inert particles: $\tau_t/\tau_v = 100$

The reduced damping curve of the acoustical wave has two distinct maxima: one for the thermal transfer $\omega_{m1} = 1/\tau_t$: $B_{m1} = 0.23$, one for the momentum transfer $\omega_{m2} = 1/\tau_v$: $B_{m2} = 0.64$. Thus the maximum reduced damping for the momentum transfer is larger than the one obtained for the thermal transfer. The sound velocity always increases with increasing values of $\omega\tau_v$. A plateau appears on the curve on an area next to ω_{m1} (figure 8).

Particles in a rarefied gas or easily deformable particles: $\tau_t/\tau_v = 1/100$

The remarks are identical to the previous case. The first maximum verifies $\omega_{m1} = 1/\tau_v$: $B_{m1} = 0.72$, this damping is due to the momentum transfer. For the second one, we have: $\omega_{m2} = 1/\tau_t$: $B_{m2} = 0.12$. Thus, the magnitude of the first maximum is larger than the magnitude of the thermal transfer. The sound speed increases with increasing values of $\omega\tau_v$ and the celerities ratio curve has got a plateau near ω_{m2} (figure 9).

CONCLUSION

The first part of this study has shown that, in most of the cases, the unsteady terms are not negligible. They may be ignored when the particles are much denser than the fluid.

The influence of the Basset term is important for intermediate values of the reduced angular frequency $\omega\tau$. It becomes negligible for small or large values of this quantity. The Basset term influence also decreases when the ratio of the mass densities χ becomes large. For small χ the history term becomes really important. That is the case of small bubbles in a liquid. Such a case, $\chi \to 0$, can hardly be observed in a gas.

The limitations of the method must be pointed out since the perturbations have been linearized. To get further in the study, we should treat concrete flow cases.

In the second part, we have studied the influences of the unsteady forces on the velocity and the damping of an acoustical wave in a two-phase flow. As the former treatment, we have made an analysis with small oscillating perturbations taking into account the compressibility of the studied medium and the thermal exchanges between the two phases. Using the dispersion equation, we show that: if γ, the isentropical coefficient of the gas, tends to unity, or β, ratio of the specific heat of the particles and the gas, tends to zero, or X, ratio of the partial mass densities of the particles and the gas, tends to zero, then the influence of the thermal relaxation decreases. The last case corresponds to a one-phase fluid, thus the velocity of sound is c_o and the damping vanishes.

For small values of $\omega\tau_v$ (equilibrium state), the velocity is smaller than the one obtained in a one-phase flow. It is also independent of the different studied combinations of the forces. This velocity does not depend on the sound frequency and the gas viscosity. The equilibrium damping is proportional to $\omega^2\rho_{ps}r_a^2/c_o\mu$, the dependence on ω^2 agrees with the Biot theory. If the particles concentration is small enough, then the damping is linearly proportional to this concentration.

For large values of $\omega\tau_v$, the results are very different. The velocity c_χ is approximately equal to c_o. The damping depends on $1/(\tau_v c_o)$, if the Basset history term is neglected. It becomes proportional to $\sqrt{(\omega/(\tau_v c_o^2))}$ when this force is taken into consideration. The Biot theory also indicates a proportionality to $\sqrt{\omega}$. If the particles concentration is small, then the damping is again linearly proportional to the value.

In the general case, a complete study of the influence of all the parameters is difficult to realize. Indeed, in our analysis, there are five nondimensional parameters: χ, ϵ, β, γ, Pr. We have chosen to study evolution tendencies for a fog at the standard temperature and pressure conditions. This concrete case shows that the velocity increases with increasing values of $\omega\tau_v$ and decreases with $1-\epsilon$. Its value is always smaller than the velocity in a one-phase flow. The reduced damping, as a function of $\omega\tau_v$, first increases and then decreases, the greatest value is obtained for $\omega\tau_v \simeq 1$. The variations, that occur around this particular point, explain the importance of particles presence in certain phenomena, such as unstabilities in rocket engines. The damping increases with increasing angular frequency and increasing particles concentration. The sound velocity increases with increasing particles radius. The damping, as a function of the particles' sizes, first increases, reaches a maximum value and finally decreases.

In the studied cases, the two relaxation times have the same magnitude. If their values are very different, we state the presence of two peaks on the reduced damping curve. One peak is due to the momentum transfer, the other is due to the thermal transfer. The amplitude of the first one is, in the present case, always larger than the amplitude of the second one. The velocity, as a function of $\omega\tau_v$, always increases and we note now the presence of a plateau in the neighborhood of $\omega\tau_t = 1$.

As was expected from the first part, the influence of the unsteady terms is not important in this case since χ is large (liquid or solid particles in a gas). The Basset force plays an important part in acoustics, effectively it makes the damping proportional to $\sqrt{\omega}$ for large angular frequencies. The sound dispersion is mainly determined by the viscous interactions between the gas and the particles. Nevertheless, the effects of heat transfer are significant and can not be neglected when the two relaxation times ratio is really different from unity.

The present method is not an exhaustive study of the unsteady terms effects. Nevertheless it gives us information and tendencies on a large range of frequencies. It might be useful for the numerician or the researcher who, most often, overlook the unstationary terms in the particles momentum equation.

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